A new mechanism for periodic bursting of the recirculation region in the flow through a sudden expansion in a circular pipe

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We report the results of a combined experimental and numerical study of specific finite-amplitude disturbances for transition to turbulence in the flow through a circular pipe with a sudden expansion. The critical amplitude thresholds for localized turbulent patch downstream of the expansion scale with the Reynolds number with a power law exponent of −2.3 for experiments and −2.8 for simulations. A new mechanism for the periodic bursting of the recirculation region is uncovered where the asymmetric recirculation flow develops a periodic dynamics: a secondary recirculation breaks the symmetry along the pipe wall and bursts into localized turbulence, which travels downstream and relaminarises. Flow visualizations show a simple flow pattern of three waves forming, growing, and bursting. *Published by AIP Publishing.* https://doi.org/10.1063/1.5022872

The flow in a circular pipe with a sudden expansion is encountered in many situations, from heat exchangers to the combustion chambers, where the turbulent flow properties are well tabulated.[1,2] Yet, the transitional regime is not well understood because it is still challenging to perform controlled experiments. Furthermore the computation requires high resolution to reproduce the details of the transition.

The flow past axisymmetric sudden expansion has previously been studied experimentally[3–5] and numerically.[6–9] The control parameter is the Reynolds number based on the inlet diameter, d, as $Re = Ud/\nu$, where U is the inlet mean flow velocity and ν is the kinematic viscosity of the fluid. Recent linear stability analysis of the 1:2 expansion[6,10] has found a critical Reynolds of several thousands before the transition takes place. Transient growth analysis[7] consistently found that infinitesimal perturbations experience maximum linear transient energy growth for Reynolds numbers of several hundreds. In practice, experiments[5] report asymmetric flow at $Re \simeq 1139 \pm 10$ and unsteady flow at $Re \simeq 1400$. This supports the idea that imperfections and the presence of finite amplitude disturbances are responsible for the transition.

Hence, this flow is considered to have sub-critical transition. Several numerical studies[8,9] have found early transition due to finite amplitude perturbations using a tilt type perturbation (along one radial direction) or a vortex type perturbation (along the axial direction with a diameter a quarter of the inlet pipe diameter). These perturbations lead to three-dimensional instabilities and localized turbulent patches. The onset of localized turbulence in shear flows can be identified through the minimum energy threshold of perturbations. For instance, in plane Couette flow, Duguet *et al.*[11] identified several instability mechanisms (Orr mechanism, oblique wave interaction, lift-up, streak bending, streak breakdown, and spanwise spreading) depending on the perturbation energy that scales as a function of $Re^{-2.7}$. For planar Poiseuille flow, Lemoult *et al.*[12] have provided a scaling law with the critical amplitude of the disturbance scaling with Re^{-1}. In the case of sudden expansion in the circular pipe, Sanmiguel-Rojas and Mullin[8] found a scaling with the critical tilt amplitude varying as $Re^{-0.006}$. One of our goals in this work is to measure and simulate a similar effect in expansion flow.

Moreover, transition to turbulence in the sub-critical regime, $1000 \lesssim Re \lesssim 4000$, has been reported to experience turbulent bursting phenomena.[3] A turbulent burst cycle is the transition between a laminar flow to a turbulent one followed by a relaminarization. A self-sustained bursting cycle in the boundary layer is related to the presence of streaks, lift-up, oscillations, and breakdown.[13] Bursts usually appear as an interaction of wave modulation or due to free-stream turbulence.[14] By constant, diameter pipe flow, bursting has been reported by Wygnanski and Champagne[15] but only in the entrance region. Here a description of quasi-periodic bursting is carried out using space-time diagrams and monitoring of the flow patterns during the cycle. The burst is due to the growth and the quasi-periodic breakdown of the underneath recirculation that initiates the localized turbulent patch.

In this letter, we first describe the numerical and experimental setups. Then, the thresholds for localized turbulence are provided for both experimental and numerical setups leading to scaling laws with exponents of −2.3 to −2.8, respectively. The spatio-temporal quasi-periodic dynamics is described and explained through the observation of the quasi-periodic growth and breakup of the recirculation region.

The Navier-Stokes equations together with the continuity equation are solved using the spectral elements method flow solver NEK5000, developed at Argonne National Laboratory.[16] For the problem in hand, the computational domain consists of an inlet pipe with diameter, d, and a length $5d$. The outlet pipe has a diameter of $D = 2d$ and the length of $150d$ to contain three times the recirculation length at $Re = 2000$. The results presented in this paper have been obtained with a mesh that has 62 300 spectral elements with the polynomial order of 5. This gives the total number of 7.9×10^6 Gauss-Legendre-Lobatto points. At the inlet: $x = -5d$, a vortex perturbation[9] of radius $r_\Omega = d/4$ is superimposed to the fully developed analytical laminar Hagen-Poiseuille velocity profile

$$\mathbf{u}_{inlet} = 2\left[1 - 4\,(r/d)^2\right]\mathbf{e}_x + \mathcal{A}\Omega\mathbf{e}_r, \tag{1}$$

where $\mathcal{A}$ is the amplitude of the vortex perturbation and Ω is the cross section velocity that is maximum at the center of the vortex and null for $r_\Omega \geq d/4$. The center of the vortex perturbation is shifted in order to break the flow symmetry. As the flow develops in the inlet section, the perturbation diffuses and becomes smoother along the inlet. At the expansion section: $x = 0$, the perturbations can be amplified depending on the values of $\mathcal{A}$ and *Re*. The outflow and wall boundary conditions are a Neumann boundary for the pressure outlet and zero-velocity at the walls, respectively.

The experiment consists of a vertical pipe divided in an inlet section with a diameter d = 1 cm and an outlet section of diameter $D = 2\ d$ after the sudden expansion. The pipe is made of 15 d long push-fitted tubes. The inlet section is 52.5 d long and the outlet section is 97.5 d long. The flow is pressure driven by a 50-liter head tank with a water level maintained constant by a spillway and the constant supply of water from a pump. Disturbances are damped by a honeycomb mesh placed inside the tank, and the connection between the tank and the pipe has a trumpet shape. The flow rate is controlled by two fine adjustment valves whose apertures are tuned using a graduated screw. The long-term temperature stability of the laboratory was controlled to ±1 °C at a mean temperature of 20 °C. By these means, it was allowed to set the Reynolds number up to 2000 ± 1%. To distinguish turbulent flow from laminar, Kalliroscope flakes are added in the water and a vertical sheet of light is arranged along the length of the pipe. Without perturbations, the flow after the expansion remains laminar for $Re \lesssim 1050$.

The inlet velocity profile strongly affects the flow downstream of the expansion.[3] Hence the inlet velocity is measured using particle image velocimetry (PIV). Acquisitions were performed with a camera (Hisense Neo). The sensor of 2560 × 2060 pixels combined with a zoom (Sigma) of focal 28-300 mm allows a resolution of 25 μm/pixel. The flow is seeded with 9 ± 2 μm hollow-glass spheres with a density of 1.1 g/ml and lightened by a laser (DualPower 135-15). The vector fields are computed from Dantec Dynamics software. Both inlet velocity profiles and the recirculation length downstream of the expansion are measured as a function of the Reynolds numbers (not shown). As *Re* increases, the recirculation axial length also increases. In a circular pipe, the recirculation is axisymmetric and pointed in the downstream direction. Numerical simulation with zero disturbances indicate that the recirculation region will reach long lengths,[7,17] following $L/d = 0.0438Re$, so the study of the flow requires long pipes and long computation domains to accommodate the oscillation of the recirculation region and the recovery of the laminar flow at the outlet.

The flow after the axisymmetric sudden expansion undergoes sub-critical transitions when disturbances are introduced.[7] To produce disturbances, a constant flow jet is applied for 90 s through a hole of diameter 3 mm connected to a syringe pump placed 5d before the expansion.

The scaling parameter is the velocity ratio, $V_r = U_j/U$, between the mean velocity of injected fluid U_j and the bulk velocity in the inlet pipe U. At a given *Re*, a minimal V_r is required to generate turbulent patches. Below this critical V_r value, disturbances will decay and the flow remains laminar. This threshold is searched experimentally for several *Re* numbers and plotted as a function of *Re* in Fig. 1(a). As *Re* increases, the flow becomes more sensitive to the disturbance and the critical V_r value decreases with a power law scale of $V_r \propto Re^{-2.3\pm0.3}$, determined by a least-square fit. For $Re < 400$, disturbances decay as they get through the expansion thus the flow remains laminar. This is the same as in the straight pipe flow, where at low *Re*, a lower bound was found.[18] The same test was done numerically to find the threshold amplitude $\mathcal{A}$ for vortex perturbations as a function of *Re*, also shown in Fig. 1(b). The least-square fit of the numerical data indicates: $\mathcal{A} \propto Re^{-2.8}$, which agrees with the experimental data but for $1000 \lesssim Re \lesssim 2000$.

Both experimental and numerical studies give similar scaling which is different from Sanmiguel-Rojas and Mullin[8] who founded a scaling $Re^{-0.006}$. The different scaling can be explained because the tilt perturbation applied at the inlet pipe wall by the authors is unphysical and different from the disturbance used here. In the present study, the perturbation is set at $-5d$ from the expansion, which means the perturbation has started to be damped by the flow when it reached the expansion.[9] Thus the flow's sensitiveness to perturbation as a function of *Re* is lower in the present cases than in the one from Sanmiguel-Rojas and Mullin.[8] A similar observation is made in stenotic flow[19] where the contraction of the flow tends to damp perturbations.

The minimal energy growth required to trigger localized turbulence was formulated in various ways.[20] Cantwell *et al.*[7] have shown that the expansion pipe flow is a noise amplifier; thus, a disturbance might grow when it gets through the expansion. Several theoretical,[21] numerical,[22] and experimental[23,24] studies found that the minimum threshold energy/amplitude to trigger disturbances scales with the Reynolds number as a power-law with an exponent from −3 down to −1. These results border the scaling obtained in the present study although the

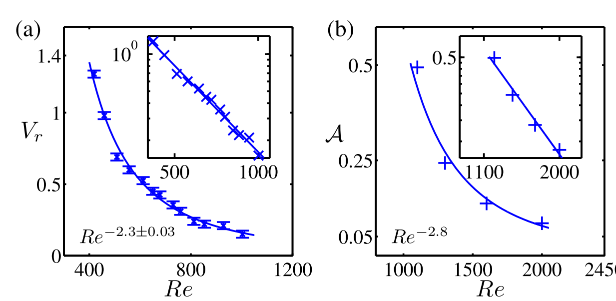

FIG. 1. (a) Experimental critical V_r versus *Re* for a crossflow jet. The continuous line is a power law fit: $Re^{-2.3\pm0.03}$. (b) The numerical critical amplitude of the vortex perturbation $\mathcal{A}$ versus *Re*. (The inset is the same results in the log-log scale.)

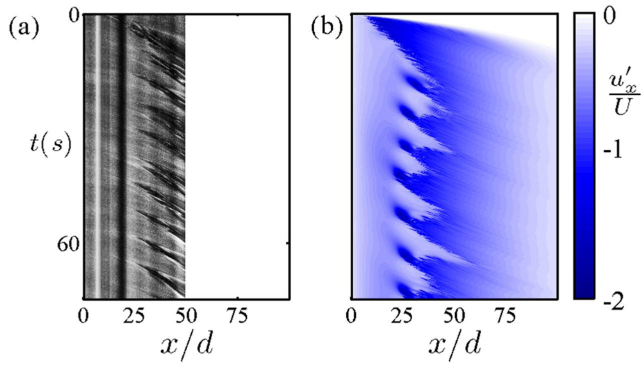

FIG. 2. Space-time diagram of the oscillating turbulent patch at (a) $Re = 700$ and $V_r = 0.35$ from experiments. The white region was not captured by the camera. (b) $Re = 1300$ and $\mathcal{A} = 0.5$ from numerical simulations, with u'_x/U being the normalized velocity fluctuation.

expansion base flow differs from the regular Poiseuille profile because of the recirculation.

The novelty of this study is the observation of quasi-periodic bursts in the transitional regime both in experiments and simulations as suggested by previous studies.[3,9] When V_r is above the critical value and a turbulent patch is formed at a fixed axial position, it is sustained as long as the injection. But when V_r is around the critical value, the flow has a different behavior. Turbulent patches are formed periodically around 20d downstream of the expansion. A space-time diagram, obtain from Kalliroscope visualizations, is presented in Fig. 2(a) for $Re = 700$ and $V_r = 0.35$ close to the critical V_r for transition. The laminar flow appears in light grey, whereas the turbulent patches are in dark. It shows that the turbulent patches are not self-sustained, they decay as new ones are formed, and they are not localized in space since they are moving downstream. The space-time diagram shows oscillations with a time period of 6.9 s, corresponding to a non-dimensional time $tU/d = 49.8$. With increasing V_r, the turbulence becomes sustained by the injected fluid and the oscillation pattern disappears.

In Fig. 2(b), numerical results are presented, specifically the streamwise velocity fluctuation $u'_x = u_x - u_b$, where u_x is the instantaneous axial velocity and u_b is the base flow velocity. Zero velocity fluctuation indicates that the velocity is at the same speed as the base flow, whereas the negative velocities indicate that the turbulent patch has a velocity reduced when compared to the base flow. Thus, an oscillatory motion is observed on the space-time diagram with a time period of 8.5 s and a non-dimensional time $tU/d = 110$, of the same order as the experiments although Re are different. Turbulent patches develop around 20d and decay as the negative velocity amplitude decreases. Again, this behavior only occurs for a vortex perturbation with an intensity near the critical value as in the experimental case.

Snapshots of the numerical axial velocities and the Kalliroscope visualizations at different stages of the bursting cycle are presented in Fig. 3. Whereas most of the previous studies reported unsteady flow,[25,26] here we provide evidence for quasi-periodic turbulent bursting. On the numerical data, the effect of the finite-amplitude vortex perturbation is to deflect the long recirculation region towards the opposite side of the disturbance as depicted on the top side of Figs. 3(a) and 3(b). Then, a secondary recirculation region appears along the wall and growths as indicted by the dimple in Figs. 3(c) and 3(d). On the experimental visualization images in Figs. 3(e)–3(i), the turbulent patch development through time appears as a wavy process with an asymmetric wave with two arms reminiscent of the symmetries of the instability modes suggested by Cantwell *et al.*[7] The visualization used here does not allow to detect the secondary recirculation region, but it is possible to detect the turbulence patch formation that takes place from the shear region. Its extent is $8 \pm 1d$. Note that the axial position of the secondary recirculation region in the numerical results and the growth of the turbulent patch from the flow visualization both appear at $x/d \approx 30$.

In order to compare sub-critical and natural transition, space-time diagrams of the axial centerline velocity are presented in Fig. 4. Two examples for natural transition are reproduced, one near the critical Re and another at higher Re. When the flow is laminar, the velocity decreases gradually, whereas

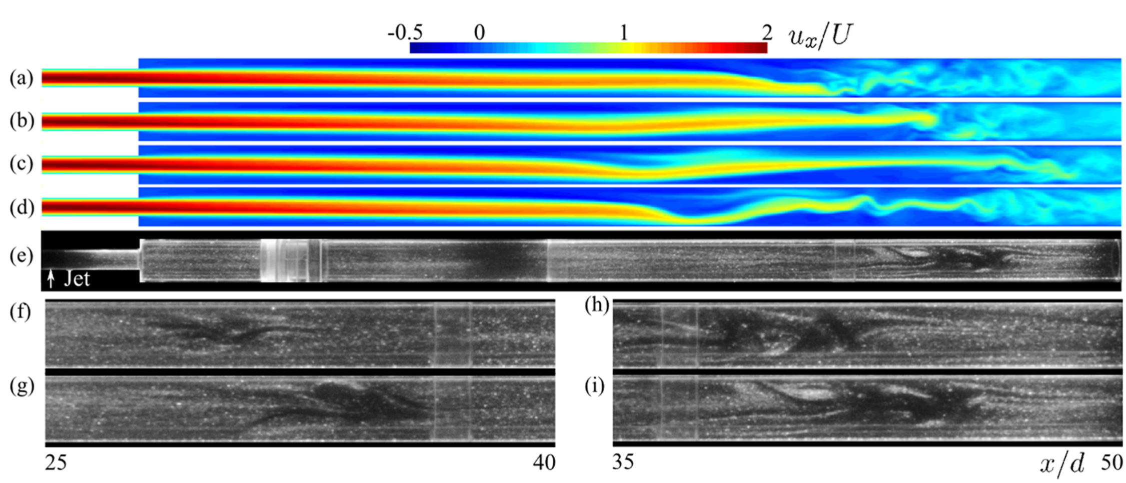

FIG. 3. [(a)-(d)] Axial velocity snapshots sequence, showing the asymmetry growth, the burst, and the translation downstream for $Re = 1300$ and $\mathcal{A} = 0.5$ at (a) $t = 8$ s, (b) 9.23 s, (c) 10 s, and (d) 10.8 s. (e) Flow visualization of the turbulent patch in the pipe at $Re = 730$ and $V_r = 0.3$ at $t = 24.1$ s from the onset of the disturbance. [(f)-(i)] Zooms on the formation in space and time of one turbulent patch for $Re = 730$ and $V_r = 0.3$ at (f) $t = 20$ s, (g) 20.8 s, (c) 22.9 s, and (d) 24.1 s (see the supplementary material for 2 videos showing the periodic bursts observed in the simulation and the experiments).

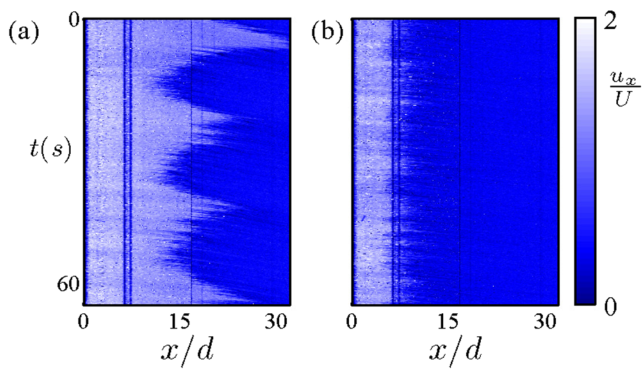

FIG. 4. Space-time diagrams of the axial velocity, u_x/U, measured using PIV. The velocity is made non-dimensional using the bulk velocity U in the inlet pipe. (a) Re = 1100 and (b) Re = 2050. The vertical line at x/d = 0 is an artifact from optical reflections. The lines at x/d = 7.5 and 30 are due to the pipe junction between the two sections, and the black vertical lines at $x/d \approx 17$ correspond to the border between the two images from the two cameras.

in the turbulent patch, the axial velocity is low and changes quickly.

In Fig. 4(a), Re = 1100, the flow exhibits an oscillation pattern as turbulent patches form downstream, propagate upstream, and decay. Although at Re = 1100 the flow also presents an oscillation between the laminar and turbulent state, the behavior of the flow is different from the sub-critical cases shown in Fig. 2. When applied, the perturbation causes the breaking of the flow and the patch extends downstream. At higher Re, i.e., Re = 2050, as depicted in Fig. 4(b), the flow becomes turbulent at $x/d \approx 10$ and no position oscillation is observed so that the turbulent patch is sustained and located as previously found in numerical simulation[9] or in the conical expansion.

Disturbed flow with long and thin recirculation region near the wall will prevent wall turbulence and lead to flow separation.[27] Indeed, downstream of the expansion, the base flow is made of a long thin recirculation region which can be made non-axisymmetric by the addition of a disturbance of sufficiently high amplitude. This leads to wavy coherent structures along the axis of the pipe. These observations suggest self-sustained motion[28] in the bulk due to vortex-wave interaction. This mechanism maintains disordered flow as long as the disturbance is applied even at low Re. Once the disturbance is removed, the decay is linear in time,[17] contrary to the uniform pipe flow close to the transition. Note that it is yet to clarify whether the eddies in the visualization may be associated with turbulence cascade and what scaling they obey.

The stability of pipe Poiseuille flow has been tested using several finite-amplitude disturbances. The critical amplitude required to cause transition scales as $Re^{-2.3\pm0.03}$ for a single crossflow jet. These results are consistent with numerical simulations that found $Re^{-2.8}$ for a vortex type perturbation. Overall, this scaling is much steeper than the scaling for the constant diameter pipe flow or plane Poiseuille flow.

Our main finding is the observation of a turbulent bursting behavior caused by the formation, growth, and breakup of a recirculation underneath the long axisymmetric recirculation due to the expansion. Using a combination of experiments and numerical simulations, we observed the formation of wavy structures that form localized turbulence. It seems a robust mechanism as it is observed clearly in experiments and numerical simulations. Yet, the range of Reynolds numbers differs and this may be due to imperfections, lack of resolution, and the choice of the disturbance that does not exactly reproduce the experiment. In the future, direct numerical simulations of the jet disturbance will be implemented in order to be faithful to the experiment.

See supplementary material for movies of the numerical simulation and the flow visualization of the bursting phenomena corresponding to Fig. 3.

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