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Segregation pattern competition in a thin rotating drum

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Results are presented of an experimental investigation into patterned size segregation of binary granular mixtures in a thin rotating drum that is half full. It is observed that streaks of small particles are formed within regions of large ones where the integer number of streaks is fixed over a range of rotation rate of the drum. Different patterns form in adjacent parameter ranges and the dynamics associated with the exchange between neighboring states is analyzed using angular spatiotemporal diagrams. These help to reveal properties of the merging mechanism for streaks of small particles. We report experimental evidence that the merging of streaks is mediated by the movement of a surplus material in a direction opposite to that of the rotation of the drum. The excess material is distributed throughout the pattern and the extra streak eventually disappears.

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I. INTRODUCTION

When a mixture of particles with different properties is caused to flow, segregation or demixing into the constituent components of the granular sample can occur [1–5]. This can have unwanted consequences for industrial processes where the objective is usually to obtain uniform mixtures. However, in the physics community, segregation is recognized as an interesting example of spontaneous pattern formation. The fundamental mechanisms underpinning segregation are not understood since differences in size, shape, surface roughness, density, etc. may each play a role.

A popular system for studying segregation is a long horizontal rotating drum where the different particles are observed to separate into bands aligned vertically to the axis of the cylinder [6–8]. This axial segregation is thought to be closely related to instabilities of a central segregated core of grains which is formed in the radial direction during the initial period of rotation of the cylinder [8–10]. Radial segregation itself is still far from understood and one way to isolate and explore it is to use a thin rotating drum where axial segregation is suppressed. In addition, thin drums allow direct observation of the majority of particles involved in the segregation and this has considerable advantages when performing detailed image analysis [11].

Two kinds of segregation pattern have been reported for thin rotating drums, viz., a central core and streaks. Photographs of core and streak patterns are displayed in Figs. 1(a) and 1(f), respectively. At modest rotation rates, the main flow is primarily confined to avalanching at the surface. The particular segregation pattern which is formed depends both on the details of the particle motion within the flow and the deposition of the material onto grains beneath. Core segregation occurs in the continuous rolling regime when small particles are trapped within the surface flow beneath the larger ones [12,13]. Hence, after several rotations of the drum all the small particles are deposited near the center of the drum and the big ones are left at the outside. Segregation into streaks or petals is characterized by the formation of stripes similar to those developed during the formation of piles [14–16]. Thus, during surface avalanching, the granular mixture is fluidized with small particles passing through voids formed between the larger ones. Consequently, a thin

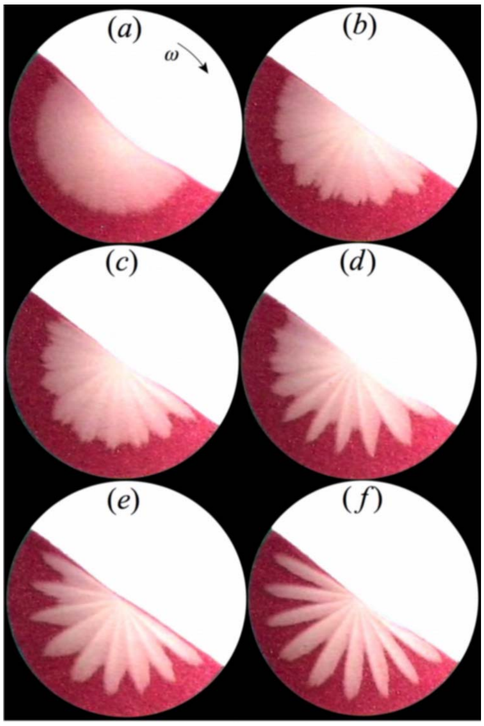

FIG. 1. (Color online) Temporal evolution of the first binary mixture of glass particles consisting on small white beads and big red (dark) beads with a relative concentration of small particles of 35%. (a) The drum rotates at 0.093 rad/s starting from core segregation. (f) Then, the system quickly evolves from the core to the streaks pattern. (b), (c), (d), (e), and (f) are snapshots of the pattern developed after 33.8, 50.7, 67.5, 84.4, and 118.2 s, respectively.

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layer of small particles is deposited at the bottom of the avalanche and a layer of big particles is segregated to the top. This mechanism is widely accepted to explain the formation of stripes in piles but it is difficult to justify for segregation in drums since the streaks may be thicker than the flowing layer. In order to explain the formation of thick stripes, a wave-breaking mechanism was proposed by Hill *et al.* [17]. This model reproduces the patterns found over a wide parameter range with the exception of the simplest case of a circular drum which is rotated at a constant speed and is half full. The reason for this exception appears to be that geometrical effects are important for pattern formation at different filling levels and container geometries [11,17–20] while they are not relevant for the special case of a half-filled cylinder.

In the case of a half-filled rotating drum, Zuriguel *et al.* [21] showed that an essential ingredient of the pattern selection process is an uphill wave comprising large particles which travel from the edge of the streaks of small particles to the center of the drum. Specifically, it was shown that two streaks are stable only if they are separated by an angular distance which allows the uphill wave to reach the center of the drum and flatten the surface. Otherwise mixing occurs giving rise to streak merging until eventually a stable configuration is reached after many drum rotations. This mechanism provides a description for the number of streaks contained within the pattern for different drum diameters, rotation frequencies, and relative concentration of small and big particles.

In many investigations [17,21], a persistent pattern is established during the first ten or so rotations of the drum. Both segregation and merging of streaks take place in this initial phase and hence it is difficult to separate the two processes. The interaction between surface segregation and pattern development is complex and probably depends on details of the particle properties. More recently, Meier *et al.* [22] obtained results for the temporal evolution of a segregated pattern in a thin rotating drum where the streaks form in the first 10–50 rotations and coarsen in a slow process that takes between 1000 and 2000 rotations. Hence the time scales of the processes involved are well separated in their case.

Here we report the results of a systematic investigation of the temporal evolution of the streaks developed in a half-filled thin rotating drum. We carry out the investigations by forming sets of spatiotemporal diagrams from the segregation patterns and applying well-known tools of pattern formation analysis (see, e.g., [23]) to study the long-term dynamics of the patterns. In particular, the techniques have been used to reveal that merging results from an intermediate streak moving in the opposite direction to the drum.

II. EXPERIMENTAL METHOD

The experimental apparatus consists of a 3-mm-thick aluminum drum with a diameter $D=24.5$ cm. The front wall of the drum is made from glass, so that direct observation of the patterns is allowed. The drum is mounted vertically and held by a shaft which is mounted in bearings. It is rotated using a feedback controlled motor drive connected via a gear box and a toothed belt. The frequency of rotation, which is measured using an optical shaft encoder, is found to be constant to within 0.002 rad/s.

Two binary mixtures are used. The first is a mixture of glass particles: (0.12 ± 0.02)-mm-diameter white and (0.71 ± 0.10)-mm-diameter red (dark in photographs) beads with a relative concentration of small particles of 35%. The second mixture is a 50% mixture of (0.50 ± 0.05)-mm-diameter black glass particles and 0.7 ± 0.1 mm white sugar crystals.

The temporal evolution of the streaks is studied through video recordings which are made using a standard charge-coupled device (CCD) camera. The image sequences are stored on a computer and further analyzed by means of an image analysis program. Specifically, a spatiotemporal diagram is formed where lines of pixels such as the one displayed in Fig. 3(a) are registered at regular time intervals and stacked horizontally in such a way that time advances from the top to the bottom of the figure. The interval of time at which the lines of pixels are registered (t_m) depends on the frequency of rotation (ω) as $t_m=\Psi/\omega$, where Ψ is the total angle where the particles are deposited in the drum as defined in [21]. Provided that $\Psi\gtrsim\pi$ $(\Psi=3.3\pm0.1$ rad) and the frequencies of rotation lay between 0.1 and 0.2 rad/s, typical values of t_m are in the range from 17 to 44 s. Images of the pattern are thus registered in a stroboscopic manner at a sample rate of t_m. This method enables us to separate the movement of the particles resulting from surface avalanches from the rotational motion of the bulk since, in the latter case, particles do not move with respect to each other. Thus the spatiotemporal diagram contains straight vertical bands when the pattern is persistent. An example of such a persistent or stable pattern can be seen in Fig. 3 for times greater than 1500 s. This corresponds to the stable pattern displayed in Fig. 3(f). A slight disadvantage of this method is that the line of pixels represented in the spatiotemporal diagram is registered only at a given distance to the center. Hence the evolution of the streaks in the radial direction is not always faithfully captured as, on occasions, apparent discontinuities can appear in the initial stages. An example of this can be seen in Fig. 3 immediately after $t=500$ s where a small increase in the radial size of the streak produces the sudden appearance of a structure in the spatiotemporal diagram.

III. STREAK MERGING

In order to separate the streak merging process from the segregation that gives rise to the formation of streaks, we use three different experimental protocols. The first involves selecting a frequency of rotation close to a value which corresponds to a change in the number of streaks. As shown in Fig. 2, the number of streaks which comprise a stable configuration depends on ω [21]. Clearly, if the drum is rotated in the ranges where changes in the number of streaks occur, the chance of obtaining a mixed intermediate state is high. This typically means that the intermediate pattern contains one more streak than the final stable configuration. The procedure allows the study of streak merging as the intermediate pattern evolves to the stable state by reduction in the number

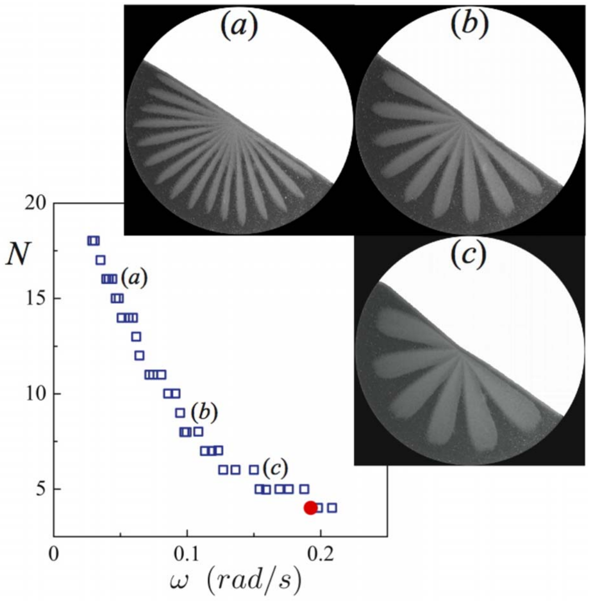

FIG. 2. (Color online) Number of streaks in the final stable pattern (N) versus the frequency of rotation, ω, for the first mixture. The full circle symbol indicates the frequency at which the first experimental protocol is carried. Photographs of the patterns of (a) 16 streaks, (b) 9 streaks, and (c) 6 streaks.

of streaks. In the example shown in Fig. 3, the selected frequency is $\omega=0.188$ rad/s which corresponds to a pattern of four streaks. However, at a frequency of $\omega=0.183$ rad/s, five streaks are found to be stable after more than 24 h of rotation (2500 rotations).

The temporal evolution of the pattern is displayed using the spatiotemporal diagram in Fig. 3. The experiment is performed by first rotating the drum at a high frequency (0.6 rad/s) in order to initiate the process from a core segregated pattern as the one shown in Fig. 3(a). At a selected time ($t=0$), a step change in rotation frequency is made to $\omega=0.188$ rad/s and the streaks start to form. The pattern evolution can be seen in the spatiotemporal diagram. At $t=0$ the spatiotemporal is noisy because the line used to build the diagram is on the edge of the segregated core. After a few rotations, streaks develop as shown in Figs. 3(b) and 3(c) and structure becomes apparent in the spatiotemporal diagram. After 600 s (15 rotations of the drum) four big streaks and a small one develop forming the pattern shown in Fig. 3(d). This pattern is shown in the spatiotemporal diagram by five white stripes. During the next 500 s (from $t=600$ to 1100 s), it may be seen in the spatiotemporal diagram that the thinnest white stripe moves from left to right, i.e., the small streak is displaced in a sense opposite to the rotation of the drum. During this period the drum has performed 15 rotations, which means that every particle in the system has avalanched down the surface 30 times. Assuming that the small streak has traveled around π rad (the position of the small streak at 600 s is approximately the same at 1100 s after a whole rotation cycle), we can estimate the angle that the small streak rotates in each avalanche which is approximately 0.1 rad.

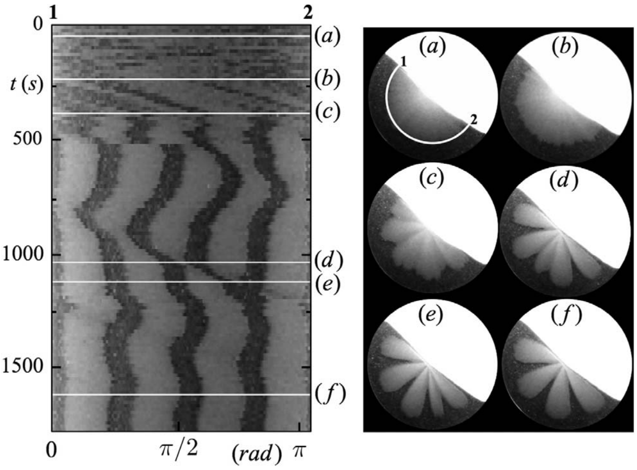

FIG. 3. Spatiotemporal diagram (left) and snapshot photographs (right) of the pattern developed by the first binary mixture rotated at 0.188 rad/s (corresponding to the full circle symbol in Fig. 2). The spatiotemporal diagram is formed by plotting the gray level at the angular position indicated by points 1 and 2 in image (a) as a function of time. Intermediate states, indicated by a horizontal white line on the diagram, are depicted on the right side of the figure: (a) core, (b) and (c) streaks development, (d) and (e) streaks merging, and (f) final four streaks state.

Careful inspection of the spatiotemporal diagram shown in Fig. 3 indicates that the thin white stripe narrows as it moves from left to right. This narrowing suggests that the small streak diminishes as small particles are continually deposited in the rest of the streaks. When the small streak eventually disappears a stable configuration is achieved and the stripes in the spatiotemporal diagram become straight. Note that in this final state, slow small oscillations of the whole system are present. The origin of this behavior is unclear and will be studied in the future.

The second experimental protocol involves rotating the drum at a set frequency and, once a stable pattern is formed, suddenly increasing the frequency which generally has the effect of reducing the number of streaks as shown in Fig. 2. This protocol is used for several different initial and final frequencies with the same qualitative results. The spatiotemporal diagram corresponding to the case of an initial rotation frequency of 0.075 rad/s, which has 11 streaks, and a final frequency of 0.14 rad/s, which corresponds to six streaks, is shown in Fig. 4. Some streaks clearly grow at the expense of others which narrow as they moved in the opposite sense to that of the rotation of the drum (from left to right in the spatiotemporal diagram). As in the first experimental protocol, the final stable pattern becomes clear in the spatiotemporal diagram when all the stripes are straight.

It can be seen in the spatiotemporal diagram shown in Fig. 4 that the process of streak coalescence takes place over different time scales. Three streaks merge 350 s (eight rotations) after the increase in the frequency to form an intermediate pattern with eight streaks. Subsequently, two further events take place and the final pattern does not appear until 1700 s (38 rotations) after the initial increase in the frequency. This behavior is in good agreement with the results presented recently in [22] where the number of streaks was proved to decrease quickly during the first rotations and

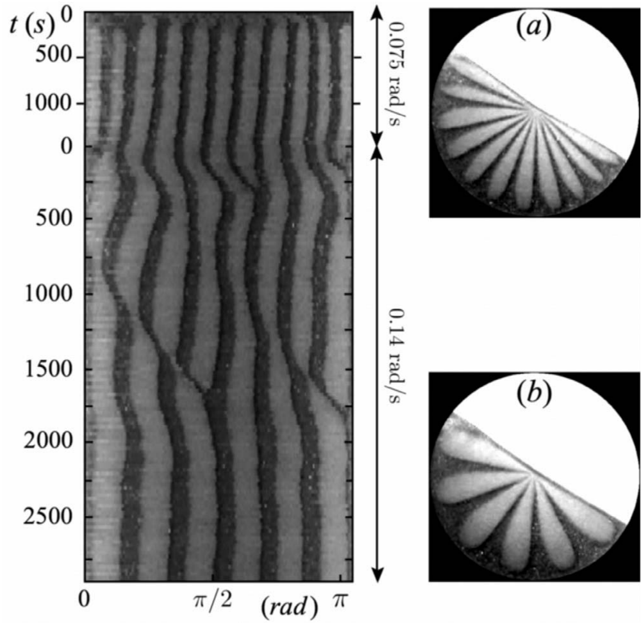

FIG. 4. Diagram (left) and snapshot photographs (right) of the first mixture. The drum rotates first at 0.075 rad/s for 1500 s and then at 0.14 rad/s as indicated by arrows in the left of the diagram. (a) is a photograph of the 11 streaks state before changing the velocity of rotation and (b) is a photograph of the six streaks final state

rather slowly when the number of streaks in the pattern was close to the number in the final configuration.

An estimate of the average angle that a small streak moves through in each avalanche can also be obtained from the spatiotemporal diagram. By way of an example we focus on the small streak represented by the white stripe at left-hand edge of the spatiotemporal diagram at $t=750$ s. This streak travels to the right covering a distance of approximately $\pi/2$ rad during the period from 750 to 1650 s after increasing the frequency. During this period the drum performs 20 rotations, so that all particles take part in 40 avalanches. We can conclude that the streak is displaced around 0.04 rad per avalanche which is approximately half that obtained with the first experimental protocol.

The third set of experiments is performed using the glass beads and sugar crystals mixture initiated from a core segregated pattern as in Fig. 5(a) where the larger white sugar particles lie around the perimeter of the drum. The drum is set rotating at a fixed frequency and it is found that the initial streak formation during avalanching layer is considerably quicker than coalescence of streaks. This can be seen in Fig. 5(b) where many small streaks appear on the edge of the core. Hence segregation and coalescence of streaks occur on different time scales and can be studied separately. The temporal evolution of the pattern is presented in Fig. 5. As with previous experimental protocols, the initial segment of the spatiotemporal diagram is noisy since the line of pixels used is located on the edge of the mixed region [Fig. 5(a)]. Subsequent rotations produce approximately 14 streaks which are underdeveloped and have different sizes [Fig. 5(b)]. At later times, streaks merge until a configuration with eight large streaks and a small one is obtained [Fig. 5(c)].

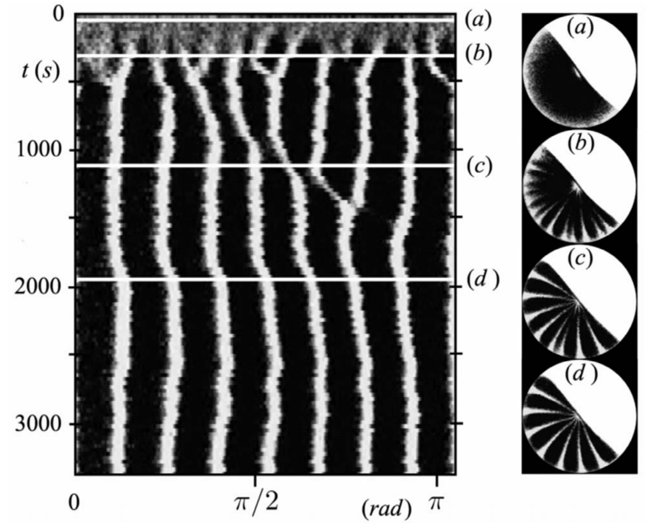

FIG. 5. (a) Diagram (left) and snapshot photographs (right) of the second mixture rotated at 0.104 rad/s from core segregation. (b) is a photograph of streaks development, (c) is a photograph of streaks merging, and (d) is the final eight streaks state.

As in the previous experiments the small streak is displaced in a sense opposite to the rotation of the drum and slowly thins as particles are distributed among the other streaks. When the small streak eventually disappears [Fig. 5(d)] the pattern is stable and the position of the streaks is fixed with respect to the drum. This spatiotemporal diagram supports the observation above that the time coalescence of streaks depends on the number of streaks in the pattern. In the initial phase, five streaks merge in less than 500 s (eight rotations of the drum). However, the remaining streak requires 16 further rotations before merging. The average angle rotated by this streak in each avalanche is approximately 0.05 rad.

IV. CONCLUSIONS

We use pattern formation analysis to study the evolution of radial segregated patterns in granular mixtures rotated in thin drums. The use of angular spatiotemporal diagrams has been implemented to uncover important aspects of the dynamics of the streaks. The streak merging in all the three experimental protocols displays the same qualitative features during pattern evolution. The streak that will eventually disappear is displaced in the sense contrary to the rotation of the drum while it decreases its size as it leaves particles in the other stable streaks. The average displacement of the small streak per surface avalanche takes different values depending on the experimental protocol. We believe that the kind of grains in the mixture, the fraction of fines, the size of the drum, and its velocity of rotation will presumably affect this displacement. This will be investigated in further experiments.

We propose that the mechanism behind the merging process shown in this work is based on the presence of the uphill wave of big particles that travels from the end of the streaks of small particles to the center of the drum flattening the surface [21]. For a given frequency, streaks arise in the way described for the stripe formation in sand piles, as a consequence of the avalanche process [14]. Hence if the distance between two streaks is large enough, in the following rotation, the uphill wave would have time to reach the center of the drum before the next streak avalanches down, and the pattern would be stable. Conversely, if the streak that is avalanching down interacts with the wave of big particles that is traveling uphill, the streak would be displaced toward the surface by the uphill wave. This results in a displacement of the streak in the direction contrary to the rotation that takes place until the distance between two consecutive streaks is large enough to allow the uphill wave to reach the center of the drum. In addition, during the interaction between the streak and the uphill wave some of the small particles pass through between the big ones with the consequent size reduction in the small streak.

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